*an Oxford specialty,' says BR, 'and bad philosophy is still philosopy!' In the idealist period the primary objective of Oxford philosophy, Russell has told me, 'was to give the place a moral tone'. The objectives of some recent capers of Oxford philosophers are hardly so clear and may even suggest the idea that, philosophically anyway, after establishing itself as the traditional home of lost causes, Oxford has finally become a lost cause itself.**

Extracted, then, from a setting not greatly different or more relevant to current thought than that in which ON DENOTING *first appeared nineteen years earlier, here is an essay in which Russell provides a succinct and eloquent summation of his point of view of some thirty years ago.*

*As a once fully matriculated member of that university, I think I can make this remark without being charged with expressing mere Cambridge prejudices.

1924

LOGICAL ATOMISM

THE PHILOSOPHY which I advocate is generally regarded as a species of realism, and accused of inconsistency because of the elements in it which seem contrary to that doctrine. For my part, I do not regard the issue between realists and their opponents as a fundamental one; I could alter my view on this issue without changing my mind as to any of the doctrines upon which I wish to lay stress. I hold that logic is what is fundamental in philosophy, and that schools should be characterized rather by their logic than by their metaphysic. My own logic is atomic, and it is this aspect upon which I should wish to lay stess. Therefore I prefer to describe my philosophy as 'logical atomism', rather than as 'realism', whether with or without some prefixed adjective.

A few words as to historical development may be useful by way of preface. I came to philosophy through mathematics, or rather through the wish to find some reason to believe in the truth of mathematics. From early youth, I had an ardent desire to believe that there can be such a thing as knowledge, combined with a great difficulty in accepting much that passes as knowledge. It seemed clear that the best chance of finding indubitable truth would be in pure mathematics, yet some of Euclid's axioms were obviously doubtful, and the infinitesimal calculus, as I was taught it, was a mass of sophisms, which I could not bring myself to regard as anything else. I saw no reason to doubt the truth of arithmetic, but I did not then know that arithmetic can be made to embrace all traditional pure mathematics. At the age of eighteen I read Mill's *Logic*, but was profoundly dissatisfied with his reasons for accepting arithmetic and geometry. I had not read Hume, but it seemed to me that pure empiricism (which I was disposed to accept) must lead to scepticism rather than to Mill's support of received scientific doctrines. At Cambridge I read Kant and Hegel, as well as Mr. Bradley's *Logic*, which influenced me profoundly. For some years I was a disciple of Mr. Bradley, but about 1898 I changed my views, largely as a result of arguments with G. E. Moore. I could no longer believe that knowing makes any difference to what is known. Also I found myself driven to pluralism. Analysis of mathematical propositions persuaded me that they could not be explained as even partial truths unless one admitted pluralism and the reality of relations. An accident led me at this time to study Leibniz, and I came to the conclusion (subsequently confirmed by Couturat's masterly researches) that many of his most characteristic opinions were due to the purely logical doctrine that every proposition has a subject and a predicate. This doctrine is one which Leibniz shares with Spinoza, Hegel, and Mr. Bradley; it seemed to me that, if it is rejected, the whole foundation for the metaphysics of all these philosophers is shattered. I therefore returned to the problem which had originally led me to philosophy, namely, the foundations of mathematics, applying to it a new logic derived largely from Peano and Frege, which proved (at least, so I believe) far more fruitful than that of traditional philosophy.

In the first place, I found that many of the stock philosophical arguments about mathematics (derived in the main from Kant) had been rendered invalid by the progress of mathematics in the meanwhile. Non-Euclidean geometry had undermined the argument of the transcendental aesthetic. Weierstrass had shown that the differential and integral calculus do not require the conception of the infinitesimal, and that, therefore, all that had been said by philosophers on such subjects as the continuity of space and time and motion must be regarded as sheer error. Cantor freed the conception of infinite number from contradiction, and thus disposed of Kant's antinomies as well as many of Hegel's. Finally Frege showed in detail how arithmetic can be deduced from pure logic, without the need of any fresh ideas or axioms, thus disproving Kant's assertion that '$7 + 5 = 12$' is synthetic—at least in the obvious interpretation of that dictum. As all these results were obtained, not by any heroic method, but by patient detailed reasoning, I began to think it probable that philosophy had erred in adopting heroic remedies for intellectual difficulties, and that solutions were to be found merely by greater care and accuracy. This view I have come to hold more and more strongly as time

went on, and it has led me to doubt whether philosophy, as a study distinct from science and possessed of a method of its own, is anything more than an unfortunate legacy from theology.

Frege's work was not final, in the first place because it applied only to arithmetic, not to other branches of mathematics; in the second place because his premises did not exclude certain contradictions to which all past systems of formal logic turned out to be liable. Dr. Whitehead and I in collaboration tried to remedy these two defects, in *Principia Mathematica*, which, however, still falls short of finality in some fundamental points (notably the axiom of reducibility). But in spite of its shortcomings I think that no one who reads this book will dispute its main contention, namely, that from certain ideas and axioms of formal logic, by the help of the logic of relations, all pure mathematics can be deduced, without any new undefined idea or unproved propositions. The technical methods of mathematical logic, as developed in this book, seem to me very powerful, and capable of providing a new instrument for the discussion of many problems that have hitherto remained subject to philosophic vagueness. Dr. Whitehead's *Concept of Nature* and *Principles of Natural Knowledge* may serve as an illustration of what I mean.

When pure mathematics is organized as a deductive system—i.e. as the set of all those propositions that can be deduced from an assigned set of premises—it becomes obvious that, if we are to believe in the truth of pure mathematics, it cannot be solely because we believe in the truth of the set of premises. Some of the premises are much less obvious than some of their consequences, and are believed chiefly because of their consequences. This will be found to be always the case when a science is arranged as a deductive system. It is not the logically simplest propositions of the system that are the most obvious, or that provide the chief part of our reasons for believing in the system. With the empirical sciences this is evident. Electro-dynamics, for example, can be concentrated into Maxwell's equations, but these equations are believed because of the observed truth of certain of their logical consequences. Exactly the same thing happens in the pure realm of logic; the logically first principles of logic—at least some of them—are to be believed, not on their own account, but on account of their consequences. The epistemological question: 'Why should

I believe this set of propositions?' is quite different from the logical question: 'What is the smallest and logically simplest group of propositions from which this set of propositions can be deduced?' Our reasons for believing logic and pure mthematics are, in part, only inductive and probable, in spite of the fact that, in their *logical* order, the propositions of logic and pure mathematics follow from the premises of logic by pure deduction. I think this point important, since errors are liable to arise from assimilating the logical to the epistemological order, and also, conversely, from assimilating the epistemological to the logical order. The only way in which work on mathematical logic throws light on the truth or falsehood of mathematics is by disproving the supposed antinomies. This shows that mathematics *may* be true. But to show that mathematics *is* true would require other methods and other considerations.

One very important heuristic maxim which Dr. Whitehead and I found, by experience, to be applicable in mathematical logic, and have since applied in various other fields, is a form of Ockham's razor. When some set of supposed entities has neat logical properties, it turns out, in a great many instances, that the supposed entities can be replaced by purely logical structures composed of entities which have not such neat properties. In that case, in interpreting a body of propositions hitherto believed to be about the supposed entities, we can substitute the logical structures without altering any of the detail of the body of propositions in question. This is an economy, because entities with neat logical properties are always inferred, and if the propositions in which they occur can be interpreted without making this inference, the ground for the inference fails, and our body of propositions is secured against the need of a doubtful step. The principle may be stated in the form: 'Wherever possible, substitute constructions out of known entities for inferences to unknown entities.'

The uses of this principle are very various, but are not intelligible in detail to those who do not know mathematical logic. The first instance I came across was what I have called 'the principle of abstraction', or 'the principle which dispenses with abstraction.'* This principle is applicable in the case of any symmetrical and transitive relation, such as equality. We are apt to infer that

* *External World*, p. 42.

such relations arise from possession of some common quality. This may or may not be true; probably it is true in some cases and not in others. But all the formal purposes of a common quality can be served by membership of the group of terms having the said relation to a given term. Take magnitude, for example. Let us suppose that we have a group of rods, all equally long. It is easy to suppose that there is a certain quality, called their length, which they all share. But all propositions in which this supposed quality occurs will retain their truth-value unchanged if, instead of 'length of the rod x' we take 'membership of the group of all those rods which are as long as x'. In various special cases—e.g. the definition of real numbers—a simpler construction is possible.

A very important example of the principle is Frege's definition of the cardinal number of a given set of terms as the class of all sets that are 'similar' to the given set—where two sets are 'similar' when there is a one-one relation whose domain is the one set and whose converse domain is the other. Thus a cardinal number is the class of all those classes which are similar to a given class. This definition leaves unchanged the truth-values of all propositions in which cardinal numbers occur, and avoids the inference to a set of entities called 'cardinal numbers', which were never needed except for the purpose of making arithmetic intelligible, and are now no longer needed for that purpose.

Perhaps even more important is the fact that classes themselves can be dispensed with by similar methods. Mathematics is full of propositions which seem to require that a class or an aggregate should be in some sense a single entity—e.g. the proposition 'the number of combinations of n things any number at a time is 2^n'. Since 2^n is always greater than n, this proposition leads to difficulties if classes are admitted because the number of classes of entities in the universe is greater than the number of entities in the universe, which would be odd if classes were some among entities. Fortunately, all the propositions in which classes appear to be mentioned can be interpreted without supposing that there are classes. This is perhaps the most important of all the applications of our principle. (See *Principia Mathematica*, *20.)

Another important example concerns what I call 'definite descriptions', i.e. such phrases as 'the even prime', 'the present King of England', 'the present King of France'. There has always been a difficulty in interpreting such propositions as 'the present King of France does not exist'. The difficulty arose through supposing that 'the present King of France' is the subject of this proposition, which made it necessary to suppose that he subsists although he does not exist. But it is difficult to attribute even subsistence to 'the round square' or 'the even prime greater than 2'. In fact, 'the round square does not subsist' is just as true as 'the present King of France does not exist'. Thus the distinction between existence and subsistence does not help us. The fact is that, when the words 'the so-and-so' occur in a proposition, there is no corresponding single constituent of the proposition, and when the proposition is fully analysed the words 'the so-and-so' have disappeared. An important consequence of the theory of descriptions is that it is meaningless to say 'A exists' unless 'A' is (or stands for) a phrase of the form 'the so-and-so'. If the so-and-so exists, and x is the so-and-so, to say 'x exists' is nonsense. Existence, in the sense in which it is ascribed to single entities, is thus removed altogether from the list of fundamentals. The ontological argument and most of its refutations are found to depend upon bad grammar. (See *Principia Mathematica*, *14.)

There are many other examples of the substitution of constructions for inferences in pure mathematics, for example, series, ordinal numbers, and real numbers. But I will pass on to the examples in physics.

Points and instants are obvious examples: Dr. Whitehead has shown how to construct them out of sets of events all of which have a finite extent and a finite duration. In relativity theory, it is not points or instants that we primarily need, but event-particles, which correspond to what, in older language, might be described as a point at an instant, or an instantaneous point. (In former days, a point of space endured throughout all time, and an instant of time pervaded all space. Now the unit that mathematical physics wants has neither spatial nor temporal extension.) Event-particles are constructed by just the same logical process by which points and instants were constructed. In such constructions, however, we are on a different plane from that of constructions in pure mathematics. The possibility of constructing an event-particle depends upon the existence of sets of events with certain properties; whether the required events exist can only be known empirically, if at all. There is therefore no *a priori* reason to expect continuity (in the mathematical sense), or to feel confident that event-particles can be constructed. If the quantum theory should seem to demand a discrete space-time, our logic is just as ready to meet its requirements as to meet those of traditional physics, which demands continuity. The question is purely empirical, and our logic is (as it ought to be) equally adapted to either alternative.

Similar considerations apply to a particle of matter, or to a piece of matter of finite size. Matter, traditionally, has two of those 'neat' properties which are the mark of a logical construction; first, that two pieces of matter cannot be at the same place at the same time; secondly, that one piece of matter cannot be in two places at the same time. Experience in the substitution of constructions for inferences makes one suspicious of anything so tidy and exact. One cannot help feeling that impenetrability is not an empirical fact, derived from observation of billiard-balls, but is something logically necessary. This feeling is wholly justified, but it could not be so if matter were not a logical construction. An immense number of occurrences coexist in any little region of space-time; when we are speaking of what is not logical construction, we find no such property as impenetrability, but, on the contrary, endless overlapping of the events in a part of space-time, however small. The reason that matter is impenetrable is because our definitions make it so. Speaking roughly, and merely so as to give a notion of how this happens, we may say that a piece of matter is all that happens in a certain track in space-time, and that we construct the tracks called bits of matter in such a way that they do not intersect. Matter is impenetrable because it is easier to state the laws of physics if we make our constructions so as to secure impenetrability. Impenetrability is a logically necessary result of definition, though the fact that such a definition is convenient is empirical. Bits of matter are not among the bricks out of which the world is built. The bricks are events, and bits of matter are portions of the structure to which we find it convenient to give separate attention.

In the philosophy of mental occurrences there are also opportunities for the application of our principle of constructions *versus* inferences. The subject, and the relation of a cognition to what is known, both have that schematic quality that arouses our suspicions. It is clear that the subject, if it is to be preserved at all, must be preserved as a construction, not as an inferred entity; the only question is whether the subject is sufficiently useful to be worth constructing. The relation of a cognition to what is known, again, cannot be a straightforward single ultimate, as I at one time believed it to be. Although I do not agree with pragmatism, I think William James was right in drawing attention to the complexity of 'knowing'. It is impossible in a general summary, such as the present, to set out the reasons for this view. But whoever has acquiesced in our principle will agree that here is prima facie a case for applying it. Most of my *Analysis of Mind* consists of applications of this principle. But as psychology is scientifically much less perfected than physics, the opportunities for applying the principle are not so good. The principle depends, for its use, upon the existence of some fairly reliable body of propositions, which are to be interpreted by the logician in such a way as to preserve their truth while minimizing the element of inference to unobserved entities. The principle therefore presupposes a moderately advanced science, in the absence of which the logician does not know what he ought to construct. Until recently, it would have seemed necessary to construct geometrical points; now it is event-particles that are wanted. In view of such a change in an advanced subject like physics, it is clear that constructions in psychology must be purely provisional.

I have been speaking hitherto of what it is *not* necessary to assume as part of the ultimate constituents of the world. But logical constructions, like all other constructions, require materials, and it is time to turn to the positive question, as to what these materials are to be. This question, however, requires as a preliminary a discussion of logic and language and their relation to what they try to represent.

The influence of language on philosophy has, I believe, been profound and almost unrecognized. If we are not to be misled by this influence, it is necessary to become conscious of it, and to ask ourselves deliberately how far it is legitimate. The subject-predicate logic, with the substance-attribute metaphysic, are a case in point. It is doubtful whether either would have been invented by people speaking a non-Aryan language; certainly they do not seem to have arisen in China, except in connexion with Buddhism, which brought Indian philosophy with it. Again, it is natural, to take a different kind of instance, to suppose that a proper name which can be used significantly stands for a single entity; we suppose that there is a certain more or less persistent being called 'Socrates', because the same name is applied to a series of occurrences which we are led to regard as appearances of this one being. As language grows more abstract, a new set of entities come into philosophy, namely, those represented by abstract words—the universals. I do not wish to maintain that there are no universals, but certainly there are many abstract words which do not stand for single universals—e.g. triangularity and rationality. In these respects language misleads us both by its vocabulary and by its syntax. We must be on our guard in both respects if our logic is not to lead to a false metaphysic.

Syntax and vocabulary have had different kinds of effects on philosophy. Vocabulary has most influence on common sense. It might be urged, conversely, that common sense produces our vocabulary. This is only partially true. A word is applied at first to things which are more or less similar, without any reflection as to whether they have any point of identity. But when once usage has fixed the objects to which the word is to be applied, common sense is influenced by the existence of the word, and tends to suppose that one word must stand for one object, which will be a universal in the case of an adjective or an abstract word. Thus the influence of vocabulary is towards a kind of platonic pluralism of things and ideas.

The influence of syntax, in the case of the Indo-European languages, is quite different. Almost any proposition can be put into a form in which it has a subject and a predicate, united by a copula. It is natural to infer that every fact has a corresponding form, and consists in the possession of a quality by a substance. This leads, of course, to monism, since the fact that there were several substances (if it were a fact) would not have the requisite form. Philosophers, as a rule, believe themselves free from this sort of influence of linguistic forms, but most of them seem to me to be mistaken in this belief. In thinking about abstract matters, the fact that the words for abstractions are no more abstract than ordinary words always makes it easier to think about the words than about what they stand for, and it is almost impossible to resist consistently the temptation to think about the words.

Those who do not succumb to the subject-predicate logic are apt to get only one step further, and admit relations of two terms, such as before-and-after, greater-and-less, right-and-left. Language lends itself to this extension of the subject-predicate logic, since we say '*A* precedes *B*', '*A* exceeds *B*', and so on. It is easy to prove that the fact expressed by a proposition of this sort cannot consist of the possession of a quality by a substance, or of the possession of two or more qualities by two or more substances. (See *Principles of Mathematics*, § 214.) The extension of the subject-predicate logic is therefore right so far as it goes, but obviously a further extension can be proved necessary by exactly similar arguments. How far it is necessary to go up the series of three-term, four-term, five-term . . . relations I do not know. But it is certainly necessary to go beyond two-term relations. In projective geometry, for example, the order of points on a line or of planes through a line requires a four-term relation.

A very unfortunate effect of the peculiarities of language is in connexion with adjectives and relations. All words are of the same logical type; a word is a class of series, of noises or shapes according as it is heard or read. But the meanings of words are of various different types; an attribute (expressed by an adjective) is of a different type from the objects to which it can be (whether truly or falsely) attributed; a relation (expressed perhaps by a preposition, perhaps by a transitive verb, perhaps in some other way) is of a different type from the terms between which it holds or does not hold. The definition of a logical type is as follows: *A* and *B* are of the same logical type if, and only if, given any fact of which *A* is a constituent, there is a corresponding fact which has *B* as a constituent, which either results by substituting *B* for *A*, or is the negation of what so results. To take an illustration, Socrates and Aristotle are of the same type, because 'Socrates was a philosopher' and 'Aristotle was a philosopher' are both facts; Socrates and Caligula are of the same type, because 'Socrates was a philosopher' and 'Caligula was not a philosopher' are both facts. To love and to kill are of the same type, because 'Plato loved Socrates' and 'Plato did not kill Socrates' are both facts. It follows formally from the definition that, when two words have meanings of different types, the relations of the words to what they mean are of different types; that is to say, there is not one relation of meaning

between words and what they stand for, but as many relations of meaning, each of a different logical type, as there are logical types among the objects for which there are words. This fact is a very potent source of error and confusion in philosophy. In particular, it has made it extraordinarily difficult to express in words any theory of relations which is logically capable of being true, because language cannot preserve the difference of type between a relation and its terms. Most of the arguments for and against the reality of relations have been vitiated through this source of confusion.

At this point, I propose to digress for a moment, and to say, as shortly as I can, what I believe about relations. My own views on the subject of relations in the past were less clear than I thought them, but were by no means the views which my critics supposed them to be. Owing to lack of clearness in my own thoughts, I was unable to convey my meaning. The subject of relations is difficult, and I am far from claiming to be now clear about it. But I think certain points are clear to me. At the time when I wrote *The Principles of Mathematics*, I had not yet seen the necessity of logical types. The doctrine of types profoundly affects logic, and I think shows what, exactly, is the valid element in the arguments of those who oppose 'external' relations. But so far from strengthening their main position, the doctrine of types leads, on the contrary, to a more complete and radical atomism than any that I conceived to be possible twenty years ago. The question of relations is one of the most important that arise in philosophy, as most other issues turn on it: monism and pluralism; the question whether anything is wholly true except the whole of truth, or wholly real except the whole of reality; idealism and realism, in some of their forms; perhaps the very existence of philosophy as a subject distinct from science and possessing a method of its own. It will serve to make my meaning clear if I take a passage in Mr. Bradley's *Essays on Truth and Reality*, not for controversial purposes, but because it raises exactly the issues that ought to be raised. But first of all I will try to state my own view, without argument.*

Certain contradictions—of which the simplest and oldest is the

* I am much indebted ty my friend Wittgenstein in this matter. See his *Tractatus Logico-Philosophicus*, Kegan Paul, 1922. I do not accept all his doctrines, but my debt to him will be obvious to those who read his book.

one about Epimenides the Cretan, who said that all Cretans were liars, which may be reduced to the man who says 'I am lying'—convinced me, after five years devoted mainly to this one question, that no solution is technically possible without the doctrine of types. In its technical form, this doctrine states merely that a word or symbol may form part of a significant proposition, and in this sense have meaning, without being always able to be substituted for another word or symbol in the same or some other proposition without producing nonsense. Stated in this way, the doctrine may seem like a truism. 'Brutus killed Caesar' is significant, but 'Killed killed Caesar' is nonsense, so that we cannot replace 'Brutus' by 'killed', although both words have meaning. This is plain common sense, but unfortunately almost all philosophy consists in an attempt to forget it. The following words, for example, by their very nature, sin against it: attribute, relation, complex, fact, truth, falsehood, not, liar, omniscience. To give a meaning to these words, we have to make a detour by way of words or symbols and the different ways in which they may mean; and even then, we usually arrive, not at one meaning, but at an infinite series of different meanings. Words, as we saw, are all of the same logical type; therefore when the meanings of two words are of different types, the relations of the two words to what they stand for are also of different types. Attribute-words and relation-words are of the same type, therefore we can say significantly 'attribute-words and relation-words have different uses'. But we cannot say significantly 'attributes are not relations'. By our definition of types, since relations are relations, the form of words 'attributes are relations' must be not false, but meaningless, and the form of words 'attributes are not relations', similarly, must be not true, but meaningless. Nevertheless, the statement 'attribute-words are not relation-words' is significant and true.

We can now tackle the question of internal and external relations, remembering that the usual formulations, on both sides, are inconsistent with the doctrine of types. I will begin with attempts to state the doctrine of external relations. It is useless to say 'terms are independent of their relations', because 'independent' is a word which means nothing. Two events may be said to be causally independent when no causal chain leads from one to the other; this happens, in the special theory of relativity, when

the separation between the events is space-like. Obviously this sense of 'independent' is irrelevant. If, when we say 'terms are independent of their relations', we mean 'two terms which have a given relation would be the same if they did not have it', that is obviously false; for, being what they are, they have the relation, and therefore whatever does not have the relation is different. If we mean—as opponents of external relations suppose us to mean—that the relation is a third term which comes between the other two terms and is somehow hooked on to them, that is obviously absurd, for in that case the relation has ceased to be a relation, and all that is truly relational is the hooking of the relation to the terms. The conception of the relation as a third term between the other two sins against the doctrine of types, and must be avoided with the utmost care.

What, then, can we mean by the doctrine of external relations? Primarily this, that a relational proposition is not, in general, logically equivalent formally to one or more subject-predicate propositions. Stated more precisely: Given a relational propositional function 'xRy', it is not in general the case that we can find predicates α, β, γ, such that, for all values of x and y, xRy is equivalent to $x\alpha$, $y\beta$, $(x, y)\gamma$ (where (x, y) stands for the whole consisting of x and y), or to any one or two of these. This, and this only, is what I mean to affirm when I assert the doctrine of external relations; and this, clearly, is at least part of what Mr. Bradley denies when he asserts the doctrine of internal relations.

In place of 'unities' or 'complexes', I prefer to speak of 'facts'. It must be understood that the word 'fact' cannot occur significantly in any position in a sentence where the word 'simple' can occur significantly, nor can a fact occur where a simple can occur. We must not say 'facts are not simples'. We can say, 'The symbol for a fact must not replace the symbol for a simple, or vice versa, if significance is to be preserved.' But it should be observed that, in this sentence, the word 'for' has different meanings on the two occasions of its use. If we are to have a language which is to safeguard us from errors as to types, the symbol for a fact must be a proposition, not a single word or letter. Facts can be asserted or denied, but cannot be named. (When I say 'facts cannot be named', this is, strictly speaking, nonsense. What can be said without falling into nonsense is: 'The symbol for a fact is not a name'.) This illustrates how meaning is a different relation for different types. The way to mean a fact is to assert it; the way to mean a simple is to name it. Obviously naming is different from asserting, and similar differences exist where more advanced types are concerned, though language has no means of expressing the differences.

There are many other matters in Mr. Bradley's examination of my views which call for reply. But as my present purpose is explanatory rather than controversial, I will pass them by, having, I hope, already said enough on the question of relations and complexes to make it clear what is the theory that I advocate. I will only add, as regards the doctrine of types, that most philosophers assume it now and then, and few would deny it, but that all (so far as I know) avoid formulating it precisely or drawing from it those deductions that are inconvenient for their systems.

I come now to some of Mr. Bradley's criticisms (*loc. cit.*, p. 280 ff.). He says:

'Mr. Russell's main position has remained to myself incomprehensible. On the one side I am led to think that he defends a strict pluralism, for which nothing is admissible beyond simple terms and external relations. On the other side Mr. Russell seems to assert emphatically, and to use throughout, ideas which such a pluralism surely must repudiate. He throughout stands upon unities which are complex and which cannot be analysed into terms and relations. These two positions to my mind are irreconcilable, since the second, as I understand it, contradicts the first flatly.'

With regard to external relations, my view is the one I have just stated, not the one commonly imputed by those who disagree. But with regard to unities, the question is more difficult. The topic is one with which language, by its very nature, is peculiarly unfitted to deal. I must beg the reader, therefore, to be indulgent if what I say is not exactly what I mean, and to try to see what I mean in spite of unavoidable linguistic obstacles to clear expression.

To begin with, I do not believe that there are complexes or unities in the same sense in which there are simples. I did believe this when I wrote *The Principles of Mathematics*, but, on account of the doctrine of types, I have since abandoned this view. To speak loosely, I regard simples and complexes as always of different types. That is to say, the statements 'There are simples' and

'There are complexes' use the words 'there are' in different senses. But if I use the words 'there are' in the sense which they have in the statement 'there are simples', then the form of words 'there are not complexes' is neither true nor false, but meaningless. This shows how difficult it is to say clearly, in ordinary language, what I want to say about complexes. In the language of mathematical logic it is much easier to say what I want to say, but much harder to induce people to understand what I mean when I say it.

When I speak of 'simples' I ought to explain that I am speaking of something not experienced as such, but known only inferentially as the limit of analysis. It is quite possible that, by greater logical skill, the need for assuming them could be avoided. A logical language will not lead to error if its simple symbols (i.e. those not having any parts that are symbols, or any significant structure) all stand for objects of some one type, even if these objects are not simple. The only drawback to such a language is that it is incapable of dealing with anything simpler than the objects which it represents by simple symbols. But I confess it seems obvious to me (as it did to Leibniz) that what is complex must be composed of simples, though the number of constituents may be infinite. It is also obvious that the logical uses of the old notion of substance (i.e. those uses which do not imply temporal duration) can only be applied, if at all, to simples; objects of other types do not have that kind of being which one associates with substances. The essence of a substance, from the symbolic point of view, is that it can only be named—in old-fashioned language, it never occurs in a proposition except as the subject or as one of the terms of a relation. If what we take to be simple is really complex, we may get into trouble by naming it, when what we ought to do is to assert it. For example, if Plato loves Socrates, there is not an entity 'Plato's love for Socrates', but only the fact that Plato loves Socrates. And in speaking of this as 'a fact', we are already making it more substantial and more of a unity than we have any right to do.

Attributes and relations, though they may be not susceptible of analysis, differ from substances by the fact that they suggest a structure, and that there can be no significant symbol which symbolizes them in isolation. All propositions in which an attribute or a relation *seems* to be the subject are only significant if they can be brought into a form in which the attribute is attributed or the relation relates. If this were not the case, there would be significant propositions in which an attribute or a relation would occupy a position appropriate to a substance, which would be contrary to the doctrine of types, and would produce contradictions. Thus the proper symbol for 'yellow' (assuming for the sake of illustration that this is an attribute) is not the single word 'yellow', but the propositional function 'x is yellow', where the structure of the symbol shows the position which the word 'yellow' must have if it is to be significant. Similarly the relation 'precedes' must not be represented by this one word, but by the symbol 'x precedes y', showing the way in which the symbol can occur significantly. (It is here assumed that values are not assigned to x and y when we are speaking of the attribute or relation itself.)

The symbol for the simplest possible kind of fact will still be of the form 'x is yellow' or 'x precedes y', only that 'x' and 'y' will be no longer undetermined variables, but names.

In addition to the fact that we do not experience simples as such, there is another obstacle to the actual creation of a correct logical language such as I have been trying to describe. This obstacle is vagueness. All our words are more or less infected with vagueness, by which I mean that it is not always clear whether they apply to a given object or not. It is of the nature of words to be more or less general, and not to apply only to a single particular, but that would not make them vague if the particulars to which they applied were a definite set. But this is never the case in practice. The defect, however, is one which it is easy to imagine removed, however difficult it may be to remove it in fact.

The purpose of the foregoing discussion of an ideal logical language (which would of course be wholly useless for daily life) is twofold: first, to prevent inferences from the nature of language to the nature of the world, which are fallacious because they depend upon the logical defects of language; secondly, to suggest, by inquiring what logic requires of a language which is to avoid contradiction, what sort of a structure we may reasonably suppose the world to have. If I am right, there is nothing in logic that can help us to decide between monism and pluralism, or between the view that there are ultimate relational facts and the view that there are none. My own decision in favour of pluralism and relations is

taken on empirical grounds, after convincing myself that the *a priori* arguments to the contrary are invalid. But I do not think these arguments can be adequately refuted without a thorough treatment of logical types, of which the above is a mere sketch.

This brings me, however, to a question of method which I believe to be very important. What are we to take as data in philosophy? What shall we regard as having the greatest likelihood of being true, and what as proper to be rejected if it conflicts with other evidence? It seems to me that science has a much greater likelihood of being true in the main than any philosophy hitherto advanced (I do not, of course, except my own). In science there are many matters about which people are agreed; in philosophy there are none. Therefore, although each proposition in a science may be false, and it is practically certain that there are some that are false, yet we shall be wise to build our philosophy upon science, because the risk of error in philosophy is pretty sure to be greater than in science. If we could hope for certainty in philosophy the matter would be otherwise, but so far as I can see such a hope would be chimerical.

Of course those philosophers whose theories, *prima facie*, run counter to science always profess to be able to interpret science so that it shall remain true on its own level, with that minor degree of truth which ought to content the humble scientist. Those who maintain a position of this sort are bound—so it seems to me—to show in detail how the interpretation is to be effected. In many cases, I believe that this would be quite impossible. I do not believe, for instance, that those who disbelieve in the reality of relations (in some such sense as that explained above) can possibly interpret those numerous parts of science which employ asymmetrical relations. Even if I could see no way of answering the objections to relations raised (for example) by Mr. Bradley, I should still think it more likely than not that some answer was possible, because I should think an error in a very subtle and abstract argument more probable than so fundamental a falsehood in science. Admitting that everything we believe ourselves to know is doubtful, it seems, nevertheless, that what we believe ourselves to know in philosophy is more doubtful than the detail of science, though perhaps not more doubtful than its most sweeping generalizations.

The question of interpretation is of importance for almost every philosophy, and I am not at all inclined to deny that many scientific results require interpretation before they can be fitted into a coherent philosophy. The maxim of 'constructions *versus* inferences' is itself a maxim of interpretation. But I think that any valid kind of interpretation ought to leave the detail unchanged, though it may give a new meaning to fundamental ideas. In practice, this means that *structure* must be preserved. And a test of this is that all the propositions of a science should remain, though new meanings may be found for their terms. A case in point, on a non-philosophical level, is the relation of the physical theory of light to our perceptions of colour. This provides different physical occurrences corresponding to different seen colours, and thus makes the structure of the physical spectrum the same as that of what we see when we look at a rainbow. Unless structure is preserved, we cannot validly speak of an interpretation. And structure is just what is destroyed by a monistic logic.

I do not mean, of course, to suggest that, in any region of science, the structure revealed at present by observation is exactly that which actually exists. On the contrary, it is in the highest degree probable that the actual structure is more fine-grained than the observed structure. This applies just as much to psychological as to physical material. It rests upon the fact that, where we perceive a difference (e.g. between two shades of colour), there is a difference, but where we do not perceive a difference it does not follow that there is not a difference. We have therefore a right, in all interpretation, to demand the preservation of observed differences, and the provision of room for hitherto unobserved differences, although we cannot say in advance what they will be, except when they can be inferentially connected with observed differences.

In science, structure is the main study. A large part of the importance of relativity comes from the fact that it has substituted a single four-dimensional manifold (space-time) for the two manifolds, three-dimensional space and one-dimensional time. This is a change of structure, and therefore has far-reaching consequences, but any change which does not involve a change of structure does not make much difference. The mathematical definition and study of structure (under the name of 'relation-numbers') form Part IV of *Principia Mathematica.*

The business of philosophy, as I conceive it, is essentially that of logical analysis, followed by logical synthesis. Philosophy is more concerned than any special science with relations of different sciences and possible conflicts between them; in particular, it cannot acquiesce in a conflict between physics and psychology, or between psychology and logic. Philosophy should be comprehensive, and should be bold in suggesting hypotheses as to the universe which science is not yet in a position to confirm or confute. But these should always be presented *as* hypotheses, not (as is too often done) as immutable certainties like the dogmas of religion. Although, moreover, comprehensive construction is part of the business of philosophy, I do not believe it is the most important part. The most important part, to my mind, consists in criticizing and clarifying notions which are apt to be regarded as fundamental and accepted uncritically. As instances I might mention: mind, matter, consciousness, knowledge, experience, causality, will, time. I believe all these notions to be inexact and approximate, essentially infected with vagueness, incapable of forming part of any exact science. Out of the original manifold of events, logical structures can be built which will have properties sufficiently like those of the above common notions to account for their prevalence, but sufficiently unlike to allow a great deal of error to creep in through their acceptance as fundamental.

I suggest the following as an outline of a possible structure of the world; it is no more than an outline, and is not offered as more than possible.

The world consists of a number, perhaps finite, perhaps infinite, of entities which have various relations to each other, and perhaps also various qualities. Each of these entities may be called an 'event'; from the point of view of old-fashioned physics, an event occupies a short finite time and a small finite amount of space, but as we are not going to have an old-fashioned space and an old-fashioned time, this statement cannot be taken at its face value. Every event has to a certain number of others a relation which may be called 'compresence'; from the point of view of physics, a collection of compresent events all occupy one small region in space-time. One example of a set of compresent events is what would be called the contents of one man's mind at one time—i.e. all his sensations, images, memories, thoughts, etc., which can coexist temporally. His visual field has, in one sense, spatial extension, but this must not be confused with the extension of physical space-time; every part of his visual field is compresent with every other part, and with the rest of 'the contents of his mind' at that time, and a collection of compresent events occupies a minimal region in space-time. There are such collections not only where there are brains, but everywhere. At any point in 'empty space', a number of stars could be photographed if a camera were introduced; we believe that light travels over the regions intermediate between its source and our eyes, and therefore something is happening in these regions. If light from a number of different sources reaches a certain minimal region in space-time, then at least one event corresponding to each of these sources exists in this minimal region, and all these events are compresent.

We will define a set of compresent events as a 'minimal region'. We find that minimal regions form a four-dimensional manifold, and that, by a little logical manipulation, we can construct from them the manifold of space-time that physics requires. We find also that, from a number of different minimal regions, we can often pick out a set of events, one from each, which are closely similar when they come from neighbouring regions, and vary from one region to another according to discoverable laws. These are the laws of the propagation of light, sound, etc. We find also that certain regions in space-time have quite peculiar properties; these are the regions which are said to be occupied by 'matter'. Such regions can be collected, by means of the laws of physics, into tracks or tubes, very much more extended in one dimension of space-time than in the other three. Such a tube constitutes the 'history' of a piece of matter; from the point of view of the piece of matter itself, the dimension in which it is most extended can be called 'time', but it is only the private time of that piece of matter, because it does not correspond exactly with the dimension in which another piece of matter is most extended. Not only is space-time very peculiar within a piece of matter, but it is also rather peculiar in its neighbourhood, growing less so as the spatio-temporal distance grows greater; the law of this peculiarity is the law of gravitation.

All kinds of matter to some extent, but some kinds of matter (viz. nervous tissue) more particularly, are liable to form 'habits', i.e. to alter their structure in a given environment in such a way that, when they are subsequently in a similar environment, they react in a new way, but if similar environments recur often, the reaction in the end becomes nearly uniform, while remaining different from the reaction on the first occasion. (When I speak of the reaction of a piece of matter to its environment, I am thinking both of the constitution of the set of compresent events of which it consists, and of the nature of the track in space-time which constitutes what we should ordinarily call its motion; these are called a 'reaction to the environment' in so far as there are laws correlating them with characteristics of the environment.) Out of habit, the peculiarities of what we call 'mind' can be constructed; a mind is a track of sets of compresent events in a region of space-time where there is matter which is peculiarly liable to form habits. The greater the liability, the more complex and organized the mind becomes. Thus a mind and a brain are not really distinct, but when we speak of a mind we are thinking chiefly of the set of compresent events in the region concerned, and of their several relations to other events forming parts of other periods in the history of the spatio-temporal tube which we are considering, whereas when we speak of a brain we are taking the set of compresent events as a whole, and considering its external relations to other sets of compresent events, also taken as wholes; in a word, we are considering the shape of the tube, not the events of which each cross-section of it is composed.

The above summary hypothesis would, of course, need to be amplified and refined in many ways in order to fit in completely with scientific facts. It is not put forward as a finished theory, but merely as a suggestion of the kind of thing that may be true. It is of course easy to imagine other hypotheses which may be true, for example, the hypothesis that there is nothing outside the series of sets of events constituting my history. I do not believe that there is any method of arriving at one sole possible hypothesis, and therefore certainty in metaphysics seems to me unattainable. In this respect I must admit that many other philosophies have the advantage, since in spite of their differences *inter se*, each arrives at certainty of its own exclusive truth.